

Spacetime dynamics and baryogenesis in braneworld

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ABSTRACT: We point out that the effective theory for the Randall-Sundrum braneworld models with bulk fields contains the baryon number violation process depending on the spacetime dynamics. Combining to the curvature-current interaction, the net baryon number observed today may be explained. The resultant baryon to entropy ratio is determined by the ratio of the Planck scales in four dimensional and five dimensional spacetime except for the parameter for CP violation.

KEYWORDS: Physics of the Early Universe, Cosmology of Theories beyond the SM.

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1. Introduction and summary

The origin of the baryon asymmetry is an important problem in cosmology [1]. Recently, stimulated by the development of string theory, the brane world idea is actively investigated. In the brane world, a new mechanism to realize baryon number violation process arises [2]. In this paper we shall point out that there exists the classical baryon number (B) violation process in the Randall-Sundrum (RS) brane world model with bulk fields. Even if they have no potentials, the baryon number conservation is violated by the spacetime dynamics via the four dimensional curvature and its derivative terms. In order to explain the observed Baryon asymmetry, we consider the CP/CPT violation via the curvature-current interaction $(\partial_\mu R)J^\mu$. This interaction is also naturally induced on the brane if there is an explicit CP violating interaction. In order to make this mechanism work, $\dot{R} \neq 0$ is necessary in radiation dominated (RD) universe. In four-dimensional general relativity $\dot{R} = 0$ in RD universe. However, interestingly, $\dot{R} \neq 0$ is realized in the RS model due to the effect of the higher dimensional gravity. Then we show that the observed baryon to entropy ratio can be explained. Our finding indicates that the higher-dimensional gravity will bring a new way to generate baryon asymmetry in the universe via the dynamics of the spacetime.

2. Baryon number violation from spacetime dynamics

We begin with the action motivated by the Randall-Sundrum(RS) model [3, 4].

$$S_{\text{tot}} = S_{\text{bulk}} + S_{\text{brane}}, \quad (2.1)$$

where S_{bulk} is the bulk action

$$S_{\text{bulk}} = \int d^5x \sqrt{-G} \left[\frac{M_5^3}{2} {}^{(5)}R(G) - \Lambda - |\nabla_M \Phi|^2 \right], \quad (2.2)$$

and S_{brane} is the brane action

$$S_{\text{brane}} = \int d^4x \sqrt{-g} \left[-\sigma + \mathcal{L}_{\text{matter}} \right]. \quad (2.3)$$

G_{MN} is the five dimensional bulk metric and $g_{\mu\nu}$ is the brane induced metric. Λ is the bulk negative cosmological constant and σ is the brane tension. We assume RS tuning and take $\Lambda = -\frac{\sigma^2}{6M_5^3}$. Φ is the bulk complex scalar field. In adS spacetimes, the scalar fields is localised on the brane as graviton.

We will derive the effective theory on the brane. Our argument relies on the braneworld holography. It is well known that the holographic picture is naturally held in RS models. The five-dimensional theory can be described by the four-dimensional theory coupled to the CFT. This is because the setup in RS models is quite resemble to that of adS/CFT correspondence [5, 6]. For example, the zero mode(φ) of the bulk complex scalar field(Φ) will be localised on the brane [7]. In our model, φ might be regarded as the squarks or sleptons that carry the Baryon/Lepton number. Applying the braneworld adS/CFT correspondence [5, 6] to the present model, we will be able to have the following effective action on the brane (See appendix A for the brief sketch of the derivation.)

$$S_{\text{eff}} \simeq \int d^4x \sqrt{-g} \left[\frac{M_4^2}{2} R + \mathcal{L}_{\text{matter}} - |\nabla\varphi|^2 - \frac{1}{4} \log \epsilon \frac{M_4^4}{M_5^6} \left(-4R_{\mu\nu} \nabla^\mu \varphi^* \nabla^\nu \varphi + \frac{4}{3} R |\nabla\varphi|^2 \right. \right. \\ \left. \left. + R_\nu^\mu R_\mu^\nu - \frac{1}{3} R^2 + \frac{2}{3} |\nabla\varphi|^4 + 2|(\nabla\varphi)^2|^2 \right) + 2|\nabla^2\varphi|^2 \right] + \Gamma_{\text{CFT}}, \quad (2.4)$$

where Γ_{CFT} is the effective action for the holographic CFT field on the brane. $g_{\mu\nu}$, R and ∇_μ are the induced metric, the Ricci scalar and the covariant derivative on the brane. M_4 is the four dimensional Planck scales given by $M_4^2 = l M_5^3$ where l is the curvature radius of the adS spacetime defined by $\Lambda = -6M_5^3/l^2$. $\mathcal{L}_{\text{matter}}$ is the leading part of the Lagrangian density for the other localised matters on the brane. The higher derivative terms come from the counter terms [5, 6]. ϵ determines the renormalization scale of the CFT.

A point is that, even if the scalar field has no potential in higher dimensional theory, $R(\partial\varphi)^2$ interaction terms can produce the B-violation process in the effective theory. Indeed, for the current $J^\mu := -i(\varphi \nabla^\mu \varphi^* - \varphi^* \nabla^\mu \varphi)$, its divergence becomes

$$\nabla_\mu J^\mu \simeq \frac{M_4^4}{M_5^6} \left[\frac{2}{3} (\nabla_\mu R) J^\mu + 4R_{\mu\nu} \nabla^\mu J^\nu \right], \quad (2.5)$$

where we assumed that the contribution of the scalar fields into the background geometry is negligible and neglected a numerical coefficient, $\frac{1}{4} \log \epsilon$. In the expanding universe, $R_{\mu\nu}$ has the time dependence in general. This means that J_μ is not conserved in general. It is reminded that there is a conserved current \tilde{J}^μ by virtue of the global $U(1)$ symmetry

$$\tilde{J}^\mu = J^\mu - \frac{M_4^4}{M_5^6} \left[4R_\nu^\mu J^\nu - \frac{4}{3} R J^\mu - \frac{4}{3} |\nabla\varphi|^2 J^\mu \right]$$

$$+4i\left(\varphi^*\nabla^\mu\varphi(\nabla\varphi^*)^2-\varphi\nabla^\mu\varphi^*(\nabla\varphi)^2\right)\Bigg]. \quad (2.6)$$

3. Gravitational baryogenesis

For the generation of the net baryon number, C and CP violations are also required [8]. Let us consider a new C and CP violation source recently proposed in paper [9](Gravitational baryogenesis). The reason why we consider is that the essence is surprisingly common with the new B-violation process found in the previous section, that is, CP violation is supposed to be raised from the interaction

$$S_{\text{int}} \sim \frac{1}{M_*^2} \int d^4x \sqrt{-g} (\partial_\mu R) J^\mu. \quad (3.1)$$

Note that there appears the same factor $\partial_\mu R$ as the B-violating terms. The authors in Ref. [9] expect such an interaction in the low energy effective theory due to some non-perturbative effects from quantum gravity. This is the simplest coupling to the spacetime curvature that breaks C and CP. In the brane world context, such a term may exist on the brane in the same way as the B-violating terms. For example, we assume that there is an explicit CP violating term on the brane as supposed in spontaneous baryogenesis [10],

$$S_{\text{int}} = \frac{1}{M} \int d^4x \sqrt{-g} (\partial_\mu \theta) J^\mu \quad (3.2)$$

where θ is a real scalar field and M is a constant with dimension of mass. Then the integration by parts and using of Eq. (2.5) bring us the following interaction term

$$S_{\text{int}} \simeq \frac{M_4^4}{M_5^6} \int d^4x \sqrt{-g} \left(\frac{-\theta}{M} \right) \left[\frac{2}{3} (\nabla_\mu R) J^\mu + 4 R_{\mu\nu} \nabla^\mu J^\nu \right]. \quad (3.3)$$

This is the curvature-current interaction that leads to gravitational baryogenesis.

Then M_* is expected to be proportional to M_5^3/M_4^2 . The coefficient depends on the model for the violation of CP, so we set $M_* = f M_5^3/M_4^2$ and leave f as a parameter in our model. If J^μ is a current which produces net $B - L$, it is not wiped away by the electroweak sphaleron process [11]. The interaction (3.1) violates CPT spontaneously due to the dynamics of the expanding universe. Thus the out of thermal equilibrium is not necessary for the baryogenesis. We denote the temperature at which the B-violating process is decoupled from thermal equilibrium as T_D . Then the produced baryon to entropy ratio will be

$$\frac{n_B}{s} \sim \frac{\dot{R}}{M_*^2 T} \Bigg|_{T_D}. \quad (3.4)$$

where n_B and s are the net baryon number density and the entropy density, respectively. An essential ingredient to realize this mechanism is a non-zero time dependence of R . In

the conventional four dimensional general relativity, $\dot{R} = 0$ in radiation dominated universe because $T_\mu^\mu = 0$, where $T_{\mu\nu}$ is the energy-momentum tensor of matters. Hence the authors in Ref. [9] considered the quantum effect, that is, trace anomaly introduced the non-zero T_μ^μ . On the other hand, in the brane world considered here, $\dot{R} \neq 0$ will be realised even in the radiation dominated universe due to the higher order curvature correction terms in the effective theory.

In the Randall-Sundrum braneworld models [3, 4], using the geometrical projection method [12], we can derive the following gravitational equation on the brane

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{M_4^2}T_{\mu\nu} + \frac{1}{M_5^6}\pi_{\mu\nu} - E_{\mu\nu}. \quad (3.5)$$

where $T_{\mu\nu}$ is the energy-momentum tensor on the brane, $\pi_{\mu\nu} := -\frac{1}{4}T_{\mu\alpha}T_\nu^\alpha + \frac{1}{12}TT_{\mu\nu} + \frac{1}{8}g_{\mu\nu}T_\beta^\alpha T_\alpha^\beta - \frac{1}{24}g_{\mu\nu}T^2$ and $E_{\mu\nu}$ is projected Weyl tensor. Here we assumed that the contribution to the gravity is dominated by the ordinal matter $\mathcal{L}_{\text{matter}}$. Hence the equation of Eq. (3.5) is equivalent with that derived from the effective action of Eq. (2.4). For example, $E_{\mu\nu}$ contains the holographic CFT stress tensor [6, 13]. Furthermore, π_μ^μ corresponds to the trace anomaly of holographic CFT fields which is evaluated by $\frac{1}{\sqrt{-g}}\frac{\delta\Gamma_{\text{CFT}}}{\delta g_{\mu\nu}}g_{\mu\nu}$. $T_{\mu\nu}$ obeys the local conservation law $\nabla^\mu T_{\mu\nu} = 0$.

From the trace term of Eq. (3.5) we obtain

$$R = -\frac{1}{M_4^2}T_\mu^\mu - \frac{1}{M_5^6}\pi_\mu^\mu \quad (3.6)$$

The second term in right-hand side is higher order corrections to conventional cosmology. Now we are thinking of the homogeneous and isotropic universe with perfect fluid. In this case R and \dot{R} become

$$R = (1 - 3w)\frac{\rho}{M_4^2} - \frac{1}{6}(1 + 3w)\frac{\rho^2}{M_5^6} \quad (3.7)$$

and

$$\dot{R} = -3(1 + w)H\left[\frac{1 - 3w}{M_4^2} - \frac{(1 + 3w)\rho}{3M_5^6}\right]\rho, \quad (3.8)$$

where $w = P/\rho$, $H = \dot{a}/a$ and a is the scale factor of the universe.

In the very early universe, the radiation is dominated and then $w = \frac{1}{3}$. So the contribution from the conventional cosmology does vanish. But, the corrections remains and then

$$\dot{R} = \frac{8}{3}\frac{H\rho^2}{M_5^6} \sim \frac{T^{10}}{M_5^6 M_4}, \quad (3.9)$$

where T is the temperature. Therefore the reaction rate is given by

$$\Gamma_B \sim \frac{M_4^4}{M_5^6}\dot{R} \sim \frac{M_4^3 T^{10}}{M_5^{12}}. \quad (3.10)$$

The decoupling temperature T_D is the temperature at which $\Gamma_B = H$, thus it is determined as

$$T_D \sim \frac{M_5^{3/2}}{M_4^{1/2}} \quad (3.11)$$

In Eq. (3.10) we tacitly assumed that the first term of right-hand side in Eq. (2.5) is dominated. It is easy to check that T_D does not depend on such an assumption. Then the net baryon to entropy ratio is given by

$$\frac{n_B}{s} \sim \frac{T_D^9}{M_*^2 M_5^6 M_4} \sim \frac{M_4^3 T_D^9}{f^2 M_5^{12}} \sim \frac{1}{f^2} \left(\frac{M_5}{M_4} \right)^{3/2}. \quad (3.12)$$

This is quite an interesting result. The resultant baryon to entropy ratio is determined by the ratio of the Planck scales in four dimensional and five dimensional spacetime except for the parameter for CP violation.

Now we consider the parameters in our model. In RS model, there is one parameter l that determines the scale below which Newton's force law is modified. Today's experiment restricts l smaller than 0.2mm. Then M_5 is bounded below, $M_5 > 10^8 \text{GeV}$. The tension of the brane is also bounded below as $\sigma > (\text{TeV})^4$. The decoupling temperature is estimated as

$$T_D \sim 10^{2.5} \left(\frac{M_5}{10^8 \text{GeV}} \right)^{3/2} \text{GeV} \quad (3.13)$$

Finally we obtain the net baryon to entropy ratio

$$\frac{n_B}{s} \sim 10^{-10} \left(\frac{0.001}{f} \right)^2 \left(\frac{10^8 \text{GeV}}{M_5} \right)^{12} \left(\frac{T_D}{10^{2.5} \text{GeV}} \right)^9. \quad (3.14)$$

This is quite reasonable value at $T_D \sim 10^{2.5} \text{GeV}$, $M_5 \sim 10^8 \text{GeV}$ and $f \sim 0.001$. As Ref. [9], we should require $T_D < T_R < M_I$ where T_R and M_I are the reheating temperature and the inflationary scale if the entropy production was occurred at the reheating after the inflation. For the chaotic inflation with the potential $V = \frac{1}{2} m^2 \phi^2$ at the very high energy, observations lead us the constraints $m \sim 5 \times 10^{-5} M_5$ and $\phi \sim 3 \times 10^2 M_5$ [14]. Thus, the inflationary scale becomes $M_I \sim V^{1/4} \sim 10^{-0.5} M_5$ which satisfies $T_D \ll M_I$.

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A. The derivation of effective action on the brane

adS/CFT correspondence in the braneworld can be formulated using the partition function in the path integral presentation [5]

$$Z = \int \mathcal{D}G \mathcal{D}\Phi e^{iS_5(G, \Phi) + \frac{i}{2}S_{\text{brane}}(g)} = \int \mathcal{D}g \mathcal{D}\varphi e^{\frac{i}{2}S_{\text{brane}} + iS_{\text{ct}} + i\Gamma_{\text{CFT}}}, \quad (\text{A.1})$$

where Γ_{CFT} is the effective action for holographic CFT on the brane. φ is the value at the location of the brane, $\varphi = \Phi|_{\text{brane}}$. S_{ct} is the counter-term which makes the total action finite. S_{ct} can be derived as the solution to the Hamilton-Jacobi(HJ) equation:

$$\begin{aligned} & -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \frac{\delta S}{\delta g_{\alpha\beta}} \left(g_{\mu\alpha} g_{\nu\beta} - \frac{1}{3} g_{\mu\nu} g_{\alpha\beta} \right) - \frac{1}{2\sqrt{-g}} \left[\left(\frac{\delta S}{\delta \phi} \right)^2 + \left(\frac{\delta S}{\delta \chi} \right)^2 \right] \\ & + \frac{1}{2} \sqrt{-g} \left[(\nabla \phi)^2 + (\nabla \chi)^2 - (R - 2\Lambda) \right] = 0. \end{aligned} \quad (\text{A.2})$$

where $\varphi = (1/\sqrt{2})(\phi + i\chi)$. HJ equation can be solved by gradient expansion with small parameter $(l/L)^2$, where l is the adS radius and L is typical length scale on the brane [15]. After all, the total effective action on the brane becomes

$$S_{\text{eff}} = S_{\text{ct}} + \frac{1}{2}S_{\text{brane}} + \Gamma_{\text{CFT}}. \quad (\text{A.3})$$

See Refs. [5, 6, 15] for the details.

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